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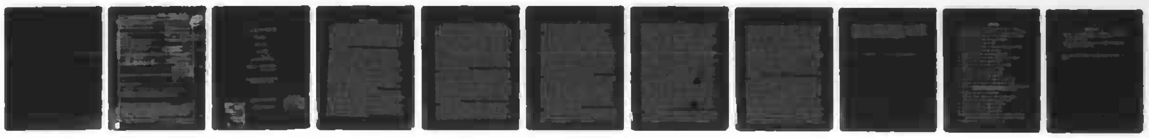

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A brief review is provided of research to incorporate more detailed information of substructure geometry, when compared to volume fraction information, in bounds on the effective property measures of composite materials. This review motivates some of the work carried out during the tenure of the grants under discussion. In addition, ~~we discuss~~ the need for a coherence function formulation of stress waves in randomly heterogeneous materials and some steps taken toward developing this formulation. are discussed

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Final Report

John J. McCoy

1 April, 1977

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Summary of Research

An appreciation of the research accomplished, with ARO support, during the past two years requires that it be viewed in the context of the longer range research efforts of the principal investigator and his colleagues. These efforts have been supported by ARO for the past five years under the broad title, "Mean Field Variations in Random Media." Perhaps an equally applicable generic title for these efforts would be, "Macroscale Response of Materials with Disordered Microstructures." The Army's interest in this research is clearly related to its interest in the development of models suitable for predicting the response of composite materials.

It is convenient in considering our research to differentiate between problems in which inertia may be neglected from those in which inertia plays a significant role. This delineation is convenient for several reasons. For example, in a statical problem the length scale on which the mean field, or macroscale, response measures vary; i.e. the L scale; is usually determined by the geometry of the specimen of interest. In a dynamical problem, on the other hand, this length scale is more properly identified with the frequency of the excitation mechanism. Further, in a statical problem, two length scales -- that on which the mean field response measures vary; i.e. the L scale; and that on which the random variations in property field measures vary; i.e. the ℓ scale -- are often sufficient to describe the experiment. In a dynamical problem, a third length scale -- the propagation range -- enters along with the mean field wavelength and the correlation lengths defined on the disordered microstructure. Thus, as might be expected, the nature of the most interesting experiments frequently changes when inertia effects become important.

It is intuitive, and has been rigorously demonstrated (1,2), that a proper theory for predicting the mean field response, in the absence of inertia effects, is an "effective modulus" theory, provided $\ell/L \ll 1$. It has also been demonstrated that gradient theories, in general, do not represent valid generalizations of the "effective modulus" theory as one moves away from the indicated limit (3). Accepting the validity of the "effective modulus" theory for the more common statical problems, the prediction problem that remains is to relate the effective moduli to suitable descriptors of the microstructure. This problem has long been of interest to the Army Research Office and a brief history of one approach to solving it will put some of our own research into perspective.

A rigorous theory for predicting the effective properties of composite materials began with the demonstrations that (i) exact prediction models based on limited information of the constituent phases; e.g. volume fraction information; cannot exist (4), and (ii) exact bounds on the desired effective property measures based on limited information of the constituent phases can usually be obtained (5). The first demonstration emphasized a fundamental fallacy in the motivations of a line of study that was quite popular among early researchers. The motivation was to improve the "law of mixtures" for estimating the effective property measure, often by performing large numbers of experiments and by choosing an improved law so as to fit the obtained data points. The second demonstration emphasized that a way does exist for constructing "well-posed" prediction models. That is, a way exists for us to ask the desired questions such that a single and unique answer is obtained. It may seem unfortunate that the manner of asking the desired question cannot actually predict the desired value but this is due to the nature of the underlying physics. Indeed it is very fortunate, and in a sense surprising, that we can say anything exactly about the desired value based on such limited information as volume fraction information.

Two further results of considerable impact were the development of property measure bounds that are referred to as Hashin-Shtrikman bounds (6), and the demonstration that these bounds are "best" bounds based on volume fraction information (6). The improvement, i.e. narrowing, of the Hashin-Shtrikman bounds over the previously reported bounds based on volume fraction information was not insignificant. Of more far-reaching effect, however, was the demonstration that the bounds were "best" in the sense that all points within the bounds corresponded to a real material. Thus, any improvement of the Hashin-Shtrikman bounds could only be achieved by the introduction of information of the composite substructure that is more refined than that contained in volume fraction information. The significance of the Hashin-Shtrikman bounds was slow in achieving the general, if not universal, acceptance that they are now afforded.

Since the Hashin-Shtrikman bounds are frequently too far separated to serve as prediction models, the problem then turned to how to incorporate the "more refined" geometric information required to obtain narrower bounds. One solution to this problem was provided in a procedure proposed by Beran (7), which leads to a hierarchy of bounds with succeeding sets in the hierarchy containing the more refined geometric information in the form of higher order statistical moments. For example, bounds based on three-point moments have been presented by Beran (7), by Brown (8), by Beran and Molyneux (9), and by McCoy (10,11). Bounds based on three-, four-, and five-point moments have been given by Elsayed (12).

While the theoretical significance of all of these more refined bounds appears clear, the practical significance is hindered by the complexity of the information required by them and by our unfamiliarity in dealing with statistical moments involving more than two points. In order to circumvent this difficulty, Miller (13,14) proposed to model the microstructure geometry and in so doing to obtain analytic expression for the multi-point statistical moments expressed in terms of a limited number of model parameters. The symmetric cell material proposed by Miller proved to be the most fruitful model. Applied to the three-point moment bounds, the symmetric cell material assumption results in bounds that are completely prescribed by a single model parameter, in addition to volume fraction information. Further, Miller was able to provide a simple geometric significance to this parameter; it is a shape factor. In so doing, he was able to actually draw a relationship between specified values of the shape factor and the qualitative concept that these values describe. Finally, Miller demonstrated that for a specified value of his shape factor, the bounds containing this information represented a distinct narrowing of the Hashin-Shtrikman bounds.

Elsayed (12) next applied the symmetric cell material assumption to his bounds that are based on three-, four-, and five-point statistical moments. The reduced bounds are prescribed by three shape factors (one from each of the multi-point moments) and two packing parameters (one from each of the four-, and five-point moments). In a further study (this one accomplished with ARO support under an earlier grant), Elsayed and McCoy (15) numerically studied equivalent bounds constructed for a two-dimensional geometry (appropriate for a fibrous composite) and compared their results to analogous bounds by Beran and Silnutzer (16), based on a single shape factor, and to the Hashin-Shtrikman bounds (17). The improvement was seen to be quite dramatic and suggests, strongly, the efficacy of incorporating more refined geometric information by means of multi-point statistical moments and of geometric modelling of the microstructure.

What was not understood, however, was the relationship between specified values of the packing parameters and other, perhaps more fundamental, measures of positional, or packing, information. To gain insight into this relationship was one of the tasks outlined for the time period of interest in this report. To see how to proceed, we recall that our understanding of the relationship between specified values of the shape factors and qualitative descriptions of inclusion shapes; e.g., spherical or plate-like, etc.; was obtained by comparing the behavior of the bounds containing the factors and exact solutions for a two-phase suspension of the desired property measure; calculated to order C, where C is the concentration or volume fraction of the suspending phase. It is well known that to this order, in a "well mixed" suspension, the effective property measure is independent of any positional information, depending only on inclusion shape information. (The task was made quite easy by the availability of a number of calculations, to order C, in the literature.) The obvious suggestion, for gaining insight into the packing parameters, is to carry out a similar comparison to order C^2, since positional information will enter the order C^2 term. The first task, then, is to obtain suitable exact calculation carried out to order C^2. This task proves to be one of considerable difficulty involving, as it does, the solution of a two body problem. A much less obvious difficulty, but one that is actually the cause of numerous erroneous results scattered throughout the literature, is the need to incorporate some aspects of long range interactions along with any short range interactions that are to be taken care of by the solution of the two body problem. An attempt to incorporate these long range interactions "in a natural manner" (18) results in convergence difficulties, the nature of which caused considerable confusion. In any event, at present a number of studies regarding this point have appeared, including one carried out by McCoy and Beran (19) with ARO support, and a number of calculations of effective property measures that are correct to order C^2 are available. Unfortunately, none of these results are directly applicable for our purposes due to the details of the suspensions for which they were accomplished and the nature of the symmetric cell model assumption. Thus, all of the exact calculations are for a distribution of equisized spheres, whereas the symmetric cell model requires a distribution of sphere sizes. At the end of the grant period, research was still going on to incorporate a distribution of sphere sizes into the exact calculations, in order to obtain a result that is suitable for the desired comparison. The principal investigator spent the last part of the time period covered by the grant visiting at Cambridge University, where most of the theory relative to order C^2 calculations is being carried out.

For dynamical problems, several questions can immediately be raised. For example, what is the validity of a dynamical effective modulus theory? Is it proper to use the averaged mass density in such a formulation or does one require an effective mass density? Also, what is a properly formulated extended theory -- e.g.

should one expect it to incorporate dispersion effects? .... decay effects? These questions were all considered in a number of studies carried out, with ARO support, some four or five years ago (20-22). During the past two years our interest has drifted somewhat from these questions pertaining to the mean field, or the completely coherent field as it is termed in the literature of stochastic radiation fields. The reason for this is that, unlike for the statical problem, the mean field does not tell the entire story in a wave propagation experiment and tells less and less of it as the frequency content of the signal increases. (Actually, the mean field would not appear to tell the whole story in a statical experiment. For example, one would suspect that fluctuations in the stress and strain fields about their mean values would be an important input to a failure theory. A general formalism for studying these fluctuations has been considered (23).) As an example of the information not contained in a mean field theory, we note that it does not enable us to predict the flow of energy in a dynamical experiment. It only considers the energy that remains in the completely coherent, or as it is sometimes referred to, the unscattered field. In order to make predictions of questions pertaining to energy flow, we require a theory expressed in terms of two-point moments, defined on the response field. It is to this problem that we have turned our attention.

While these last mentioned theories have received scant attention in the context of a solids problem, there exist fairly well established literatures on them in the context of electromagnetic radiation fields, or more to the point, in the context of acoustic signals in a fluid. A significant difference between waves in solids and waves in fluids is, of course, the presence of more than one propagation mode in the former and the possibility of mode conversion due to scattering. During the past two years we have considered several aspects of this problem. In one study (24), we considered the development of a "Parabolic Theory of Stress Wave Propagation Through Inhomogeneous Linearly Elastic Solids." A parabolic wave theory differs from a complete wave theory in allowing propagation only in directions of increasing range. Thus, when applicable, it is well suited to numerical computations using a range incrementing procedure. We note that the validity of the parabolic formalism would appear to be crucial for the development of the random scattering theory envisioned. In the context of this same question, a small effort was undertaken to develop experimental capability to study propagation experiments involving inhomogeneous medium. In a second study (25,26), we considered the development of a scattering theory, which is suitable for experiments with the possibility of more than one mode of propagation. While the theory is most directly applicable to acoustical waves in waveguides, the significance that it has for the solids is this presence of more than one propagation mode. Finally, a further study toward developing a theory for describing high frequency coherence experiments in randomly inhomogeneous linearly elastic solids is well underway (27).

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